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ROLE OF CAPILLARY FORCES IN FILTRATION OF A GAS–CONDENSATE MIXTURE NEAR A WELL

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The problem of steady-state flow of a gas–condensate mixture near an operating well is considered. The effect of capillary forces on condensate distribution in a porous medium is studied both analytically and numerically. It is shown that capillary forces can serve to decrease the amount of condensate, with the effect occurring in both cases — wetting and nonwetting liquid phases.

It is well known that the problem of radial steady-state filtration of a gas–condensate mixture is integrated in quadratures [1–3]. In the case of a zero capillary jump of pressure, a stationary solution can be obtained in the plane case as well [4]. The properties of exact solutions can be used for interpretation of stationary studies of gas–condensate mixtures [5]. Although the employed method of solution covers the case of nonzero capillary forces [2], the effect of the latter on the solution and, in particular, on the distribution of condensate saturation in the critical area of the well has not been studied in detail. Numerical and analytical solutions were obtained only for the case of coinciding pressures in gas and condensate [1–3]. In the present paper, we give analytical and numerical solutions of the problem of filtration of a gas–condensate mixture with capillary forces.

It will be recalled that for gas–condensate reservoirs under the formation conditions most of the porous volume is occupied by the hydrocarbon gas phase and the amount of the hydrocarbon liquid phase is small or equal to zero. As pressure decreases, formation gas liberates the liquid phase (condensate) due to retrograde condensation [6]. Condensate can be accumulated in a large amount in the critical area of the well, thus sharply decreasing its output. Therefore, it is of importance to predict the condensate distribution near the well.

We consider a steady-state, isothermal, cylindrically symmetric flow of the gas–condensate mixture in an isotropic collector. In this case, all characteristics of the mixture depend on the distance to the well axis r. The well is assumed to be vertical. The determining equations are the conditions of condensation of the components:

$$\partial_r \left(r \left(n_{ig} u_{\mathrm{g}}^r + n_{ic} u_{\mathrm{c}}^r\right)\right) = 0 \, . \tag{1}$$

Here and below, i and j take values 1, ..., M, thus corresponding to the number of the component. For the rates of filtration we take the Darcy law

$$u_{\mathrm{g}}^r = - k f_{\mathrm{g}} \mu_{\mathrm{g}}^{-1} \partial_r p_{\mathrm{g}} \, , \quad u_{\mathrm{c}}^r = - k f_{\mathrm{c}} \mu_{\mathrm{c}}^{-1} \partial_r p_{\mathrm{c}} \, . \tag{2}$$

We assume that all properties and characteristics of the gas and condensate are determined by expressions which are the same for both phases [6]. In this case, since flow is isothermal, the temperature dependence will be omitted. Thus, $p_{\mathrm{g}} = p(n_{i\mathrm{g}})$, $p_{\mathrm{c}} = p(n_{i\mathrm{c}})$, $\mu_{\mathrm{g}} = \mu(n_{i\mathrm{g}})$, $\mu_{\mathrm{c}} = \mu(n_{i\mathrm{c}})$, $\kappa_{i\mathrm{g}} = \kappa_i(n_{j\mathrm{g}})$, and $\kappa_{i\mathrm{c}} = \kappa_i(n_{j\mathrm{c}})$.

The relative phase permeabilities of the gas and condensate are functions of the condensate saturation s_{c}: $f_{\mathrm{g}} = f_{\mathrm{g}}(s_{\mathrm{c}})$, $f_{\mathrm{c}} = f_{\mathrm{c}}(s_{\mathrm{c}})$.

The densities of the components in the gas and condensate are related by the conditions of local thermodynamics equilibrium:

$$\kappa_{i\mathrm{g}} = \kappa_{i\mathrm{c}} \, , \tag{3}$$

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$$p_{\mathrm{g}} - p_{\mathrm{c}} = p_{\mathrm{cap}} \, . \tag{4}$$

Here $p_{\mathrm{cap}} = p_{\mathrm{cap}}(s_{\mathrm{c}})$ is a smooth monotonically decreasing function of the condensate saturation s_{c}. If the condensate is the wetting phase, then $p_{\mathrm{cap}} \geq 0$; if it is the nonwetting phase, then $p_{\mathrm{cap}} \leq 0$.

Equations (1) have M integrals

$$2\pi rh\,(n_{i\mathrm{g}}u^{r}_{\mathrm{g}} + n_{i\mathrm{c}}u^{r}_{\mathrm{c}})) = -\,Q_i \, . \tag{5}$$

We introduce the notation $Q = \sum\limits_{i=1}^{M} Q_i$, $n_{\mathrm{g}} = \sum\limits_{i=1}^{M} n_{i\mathrm{g}}$, $n_{\mathrm{c}} = \sum\limits_{i=1}^{M} n_{i\mathrm{c}}$, $c_{i\mathrm{g}} = n_{i\mathrm{g}}/n_{\mathrm{g}}$, $c_{i\mathrm{c}} = n_{i\mathrm{c}}/n_{\mathrm{c}}$.

From Eqs. (5) follows a relation that relates the concentrations in the phases:

$$(1 - W)\, c_{i\mathrm{g}} + Wc_{i\mathrm{c}} = c_{i0} \, , \tag{6}$$

$$W = -\,2\pi rhn_{\mathrm{c}}u^{r}_{\mathrm{c}}/Q = 1 + 2\pi rhn_{\mathrm{g}}u^{r}_{\mathrm{g}}/Q \, . \tag{7}$$

The system of equations (3), (4), and (6) can be interpreted as a description of disintegration of the mixture with a composition c_{i0} into gas and condensate with compositions $c_{i\mathrm{g}}$ and $c_{i\mathrm{c}}$ and pressures in the phases p_{g} and p_{c}, respectively. We note that for gas–condensate fields the situation is typical when at a distance from the operating well condensate is either absent in the formation or its amount is insufficient and the mobility of the condensate can be neglected. Here, as follows from relations (6), (7), concentrations c_{i0} describe the gas phase of the formation mixture. In the general case, concentrations c_{i0} determine the composition of the moving part of the formation mixture. The problem of disintegration of a c_{i0} mixture into gas and condensate is independent of the problem of filtration and can be solved by the means and methods of chemical thermodynamics, for example, on the basis of the equation of state chosen for a mixture [6]. With a fixed set of c_{i0}, the solution, generally speaking, has the following functional form:

$$c_{i\mathrm{g}} = c_{i\mathrm{g}}\,(p_{\mathrm{g}}, p_{\mathrm{c}}) \, , \quad n_{\mathrm{g}} = n_{\mathrm{g}}\,(p_{\mathrm{g}}, p_{\mathrm{c}}) \, , \quad c_{i\mathrm{c}} = c_{i\mathrm{c}}\,(p_{\mathrm{g}}, p_{\mathrm{c}}) \, , \quad n_{\mathrm{c}} = n_{\mathrm{c}}\,(p_{\mathrm{g}}, p_{\mathrm{c}}) \, , \quad W = W\,(p_{\mathrm{g}}, p_{\mathrm{c}}) \, . \tag{8}$$

Instead of the spatial coordinate r, it convenient to introduced the collective variable

$$\xi = Q\,(2\pi kh)^{-1} \ln\,(r/a) \, . \tag{9}$$

Then, from Eqs. (2), (7), and (9) the system of ordinary differential equations for pressures in the phases follows:

$$\frac{dp_{\mathrm{g}}}{d\xi} = \frac{(1 - W)\,\mu_{\mathrm{g}}}{n_{\mathrm{g}}f_{\mathrm{g}}} \, , \tag{10}$$

$$\frac{dp_{\mathrm{c}}}{d\xi} = \frac{W\mu_{\mathrm{c}}}{n_{\mathrm{c}}f_{\mathrm{c}}} \, . \tag{11}$$

With account for expressions (8), the right-hand sides of Eqs. (10), (11) are functions of the variables p_{g}, p_{c}, and s_{c}. The problem of determination of the functions $p_{\mathrm{g}}(\xi)$, $p_{\mathrm{c}}(\xi)$, and $s_{\mathrm{c}}(\xi)$ becomes closed if the finite (nondifferential) equation (4) is added to Eqs. (10), (11).

In view of the self-similarity of the system of equations (4), (10), and (11), one solution of the problem $p_{\mathrm{g}} = p_{\mathrm{g}}(\xi)$, $p_{\mathrm{c}} = p_{\mathrm{c}}(\xi)$, and $s_{\mathrm{c}} = s_{\mathrm{c}}(\xi)$ generates a range of other solutions $p_{\mathrm{g}} = p_{\mathrm{g}}(\xi + \xi_0)$, $p_{\mathrm{c}} = p_{\mathrm{c}}(\xi + \xi_0)$, and $s_{\mathrm{c}} = s_{\mathrm{c}}(\xi + \xi_0)$, where ξ_0 is arbitrary. Allowing for determination of the variable ξ in (9), we can draw a conclusion that one of the solutions found makes it possible to automatically obtain a solution of the initial problem of filtration (1), (3), (4) at any other value of the parameters Q, k, h, and a.

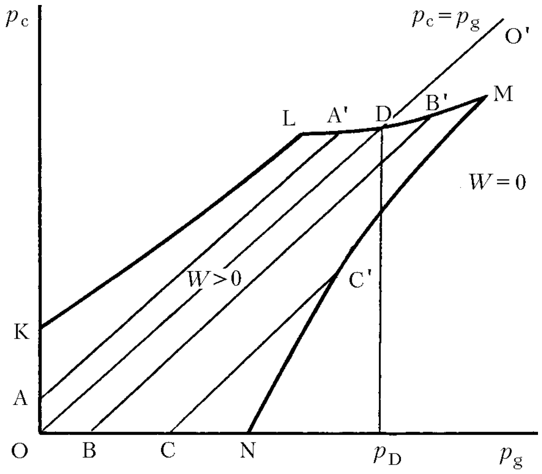

Fig. 1. Region of two-phase states of the multicomponent mixture with different pressures in the phases.

The solution of the problem of filtration of a gas–condensate mixture must satisfy ordinary boundary conditions — pressures on the well face p_w and in the formation p_0 are set:

$$p_g \big|_{r=r_w} = p_w\,, \quad p_g \big|_{r=r_0} = p_0\,. \tag{12}$$

The output of the well Q is found from solution of the problem with account for conditions (12).

It is worth noting that, generally speaking, solution of the problem of filtration is not ambiguously determined by the boundary conditions (12). Indeed, in the general case, the trajectories of the system (10), (11) continuously occupy the plane p_g, p_c and, consequently, form a parametric set. Taking any trajectory, from conditions (12) one can determine values of the parameter ξ which correspond to the radii r_w and r_0 and then, from Eq. (9), calculate the parameters Q and a. Thus, an infinite range of solutions of the problem of filtration can be found, and for selection of a singular solution one must have an additional criterion.

The behavior of solutions greatly depends on the function $W = W(p_g, p_c)$. The region $W > 0$ is qualitatively depicted in Fig. 1 in the plane (p_g, p_c). This region is bounded by the closed curve OKLMNO. The sections OK and ON are determined by the lower limit of the considered range of pressures, which is assumed to be much higher than atmospheric. The curve LM is determined by the equation $W(p_g, p_c) = 0$. In the vicinity of this curve, the function $W = W(p_g, p_c)$ is determined as smooth. The function $W = W(p_g, p_c)$ has a discontinuity: on the curves KL and MN, on one side of the curve the function $W = W(p_g, p_c)$ takes, generally speaking, finite positive values and on the other side of the curve it is exactly zero. This structure of the boundary of the region $W > 0$ is substantiated theoretically in the Appendix. The point D (see Fig. 1) corresponds to the value of pressure p_D at which the gas mixture becomes unstable in the volume (the dew point).

We now consider the problem of nonuniqueness of the solution in more detail. According to Eq. (11), the condensate is mobile ($f_c > 0$) only provided that $W > 0$. Consequently, if there is a nonzero threshold of condensate mobility $s_{c1} > 0$, then for $W > 0$ the condensate saturation must be higher than this threshold, $s_c > s_{c1}$. At $W = 0$ the condensate (if any) is motionless.

By virtue of this, two mechanisms of origination of the ambiguity can be revealed.

I. Let $s_c = s_c(p_g)$ be a value of saturation depending on pressure in the gas phase on the curve LM (see Fig. 1), which is obtained as a result of elimination of the parameter p_c from the equations $W(p_g, p_c) = 0$ and (4). In other words, $s_c = s_c(p_g)$ is such a limiting value of condensate saturation at which the mixture with composition c_{i0} cannot disintegrate in the porous medium into the gas phase with pressure p_g and the liquid phase with saturation within the range $[s_c(p_g), 1]$. Then, at certain values of pressure in the gas medium which are higher than saturation pressure, $p_g > p_D$, the inequality $s_c(p_g) < s_{c1})$ can hold. In this case, the moving gas phase with the composition c_{i0} can coexist at thermodynamic equilibrium with the motionless liquid phase, with the liquid-phase saturation lying within the range

$[s_c(p_g),\ s_{c1}]$. Despite the fact that the liquid is motionless, it affects the pressure field, since on the right-hand side of Eq. (10) the coefficient $f_g = f_g(s_c)$ is present.

The presence or absence of the motionless liquid phase is not in conflict with the main equations of the filtration problem (1), (3), and (4). Therefore, solutions with an arbitrary set of ranges of the radial coordinate r are possible, in principle, where the motionless liquid phase is present. However, if the flow near the well, which is formed as a result of gradual decrease of pressure on the well face, is considered and condensate in the formation is initially absent, the solutions with motionless liquid must be excluded. On the contrary, if motionless condensate initially existed in the formation, it must remain in the steady-state flow as well.

II. At the point $\xi = \xi_t$ of transition from the region $W = 0$ to the region $W > 0$, a jump of saturation takes place. As was discussed above, when $\xi > \xi_t$ the condensate saturation is zero or lies within the range $[s_c(p_g,\ s_{c1})$. When $\xi < \xi_t$, the condensate saturation is strictly higher than s_{c1} and simultaneously is strictly lower than the value of s_{c2}, which corresponds to the threshold of mobility for the gas. The value of $s_{c,t} = s_c\big|_{\xi=\xi_t-0}$ can be selected arbitrarily within the range $s_{c1} < s_{c,t} < s_{c2}$, and this choice unambiguously determines the solution in the region $\xi < \xi_t$.

As in case I, if a steady-state flow is formed from the state of the formation system where condensate is absent, one must choose a solution with a minimum value of the condensate saturation. This corresponds to the boundary condition

$$s_{c,t} = s_{c1}\ . \tag{13}$$

Thus, the choice of solution must be based on the pre-history of the formation system as a whole. In what follows, we consider the solution with a minimum possible amount of condensate. In the region $W = 0$, condensate is absent and the pressure field is determined by Eq. (10) with $f_g = 1$. In the region $W > 0$, the solution of the problem is determined by the system of equations (4), (10), and (11) with boundary condition (13).

The point of entry into the region $W > 0$ can be found in the plane $(p_g,\ p_c)$ as the intersection of the curves KL, LM, and MN (see Fig. 1) with the straight line

$$p_g - p_c = p_{\mathrm{cap}}\,(s_{c1})\ . \tag{14}$$

Figure 1 presents possible versions of the position of the straight line (14) in the plane (p_g, p_c): AA′ is the case where the condensate is the nonwetting phase, BB′ and CC′ are the cases where the condensate is the wetting phase, and OO′ is the case where the capillary jump is zero. For the version BB′, condensate appears at a pressure in the gas p_g exceeding the saturation pressure p_D for the versions AA′ and CC′, i.e., $p_g < p_D$.

It is pertinent to emphasize that condition (14) for the point of mixture entry into the region $W > 0$, where two mobile phases with different pressures coexist, takes place at the least possible amount of condensate behind the jump (13). Due to the nonuniqueness of the solution, jumps with a larger amount of condensate are possible.

In order to estimate the possible effect of capillary forces on a gas–condensate mixture flow, we obtained a series of numerical solutions of the problem (4), (10), and (11) for a mixture composition corresponding to the second object of the Karachaganak oil-gas-condensate field (Republic of Kazakhstan). The mixture composition (in mole fractions) is as follows: $c_{N_2} = 0.0103$, $c_{CO_2} = 0.0462$, $c_{H_2S} = 0.0432$, $c_{CH_4} = 0.6269$, $c_{C_2H_6} = 0.0822$, $c_{C_3H_8} = 0.0308$, $c_{nC_4H_{10}} = 0.0062$, $c_{iC_4H_{10}} = 0.0103$, $c_{C_5} = 0.0285$, $c_{C_6} = 0.0149$, and $c_{C_{7+}} = 0.1005$.

The initial formation pressure of 540 bars is close to the saturation pressure of 530 bars (1 bar = 10^5 Pa). Thermodynamic characteristics and phase transitions were calculated by the Peng–Robinson equation of state [6]. The viscosities of the gas and condensate are taken to be constant: $\mu_g = 2.3\cdot 10^{-5}$ Pa·sec and $\mu_c = 4.9\cdot 10^{-4}$ Pa·sec, and a value of $\xi = 0$ to be correspondent to a gas pressure of 535 bars. The exponential permeabilities to phase were used:

$$f_g = (s_g - s_{g1})^a \big/ (1 - s_{g1})^a\ , \quad f_c = (s_c - s_{c1})^b \big/ (1 - s_{c1})^b\ ,$$

$$a = b = 2\ , \quad s_{g1} = 1 - s_{c2} = 0.08\ , \quad s_{c1} = 0.12\ .$$

Calculations were made for the following model expressions of the capillary pressure:

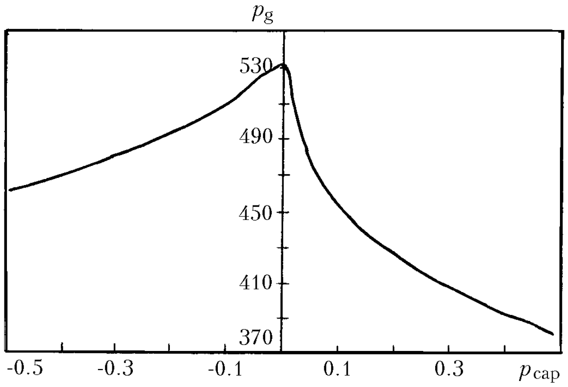

Fig. 2. Dependence of pressure in the gas on the capillary jump on the boundary of the two-phase region. p_g, p_{cap}, bar.

$$p_{cap}(s_c) = \frac{Cs_{c1}}{s_c(1-s_{c1})} - \frac{Cs_{c1}}{1-s_{c1}} \geq 0 \quad (C > 0) \quad \text{(wetting)}\,,$$

$$p_{cap}(s_c) = \frac{Cs_{g1}}{(1-s_c)\, s_{c2}} - \frac{Cs_{g1}}{s_{c2}} \leq 0 \quad (C < 0) \quad \text{(nonwetting)}\,.$$

In solution of Eq. (11), at the point of entry into the two-phase region an eliminable singularity of the 0/0 type can be encountered on the right-hand side. Indeed, it is convenient, using relation (4), to introduce a new unknown quantity φ, which monotonically depends on the condensate saturation when $s_c > c_{c1}$, instead of unknown pressure in the condensate p_c:

$$\varphi = \varphi(s_c) = \int\limits_{s_{c1}}^{s_c} p'_{cap}(z)\, f_c(z)\, dz\,. \tag{15}$$

Then, making the substitution of variables in (11) using (15), we obtain an equation which does not involve singularities:

$$\frac{d\varphi}{d\xi} = \frac{(1-W)\,\mu_g f_c}{n_g f_g} - \frac{W\mu_c}{n_c}\,. \tag{16}$$

By virtue of the monotonicity of function (15), we can exclude the dependence of the right-hand sides of Eqs. (10), (16) on the condensate saturation s_c and pressure in the condensate p_c and express them as functions of p_g and φ. Then the system (10), (16) can be solved numerically by the Runge–Kutta method.

Figure 2 gives the values of gas pressure p_g at the point of condensate occurrence as a function of capillary pressure at this point p_{cap}. Figure 3 presents the results of the calculation of pressure and saturation correspondingly for different values of the parameter C. The quantities which have dimensions of pressure, the parameter C included, are measured in bars. The output of the well Q is recalculated in units adopted in practice in the development of gas-condensate fields, viz., ths. m^3/day for normal conditions, the coefficient of permeability k is given in millidarcies (1 mD = 10^{-15} m^2), and the formation thickness h is given in meters.

Analysis of the graphs obtained shows that a larger absolute value of the capillary jump facilitates smaller values of the parameter ξ at the point of occurrence of condensate. This indicates a decrease of the condensate plug at a fixed output, thickness, and permeability. Simultaneously, the pressure in the gas phase, which corresponds to the point of condensate occurrence, can decrease (see Fig. 2). The region of capillary jumps, where the gas pressure at the point of condensate precipitation is higher than the pressure of saturation in the volume (section DM in Fig. 1), turns out to be very small. Although the size of the condensate plug can strongly differ for various solutions, qualitatively

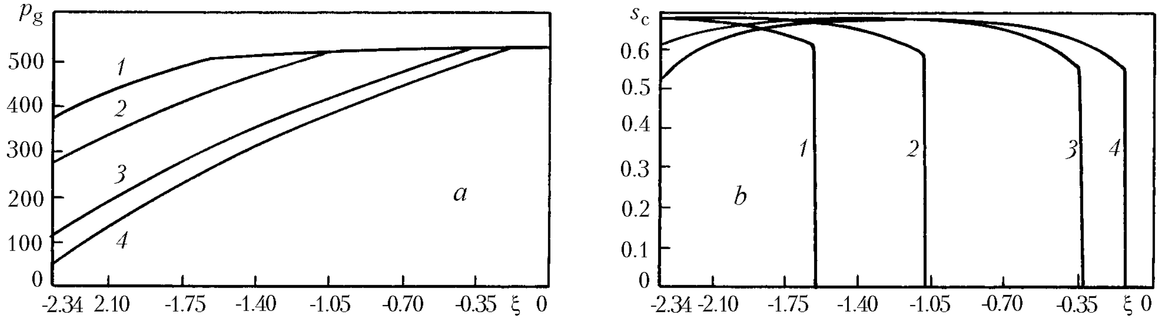

Fig. 3. Pressure in the gas (a) and saturation of the condensate (b) for gas–condensate mixture flows with different capillary forces: 1) $C = 0.017$; 2) $C = -0.1$; 3) $C = -0.05$; 4) $C = 0.014$. p_g, bars; ξ, ths. $m^3/(\text{day}\cdot\text{mD}\cdot\text{m})$.

the distribution of condensate in the critical zone of the well appears to be the same: the condensate saturation increases sharply behind the jump, thus reaching a value of 0.5–0.7 (see Fig. 3b). In the condensate-plug region, the slope of the pressure curve as a function of the parameter ξ changes, with the slope of the curves being almost the same for various solutions.

Thus, the problem of gas–condensate mixture flow near the well is reduced to the system of two ordinary differential equations for pressures in the phases. The problem of phase transition is aptly separated from the problem of filtration. Numerical solutions which demonstrate a strong effect of capillary forces on the distribution of condensate in the critical zone of the well are found for a real mixture of the formation. Here it is found that in cases of both wetting and nonwetting a large absolute value of the capillary jump can lead to a decrease in the amount of precipitated condensate.

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APPENDIX

Below we present the derivation of differential relations which satisfy the functions $c_{ig} = c_{ig}(p_g, p_c)$, $c_{ic} = c_{ic}(p_g, p_c)$, and $W = W(p_g, p_c)$ (see (8)) and substantiation of the shape of the boundary of the region $W(p_g, p_c) > 0$. Summation is made by repetitive indices.

It is convenient to use the thermodynamic potential (Gibbs energy) of the mixture $\psi = \psi(p, c_i) = c_i\kappa_i$. Since the concentrations satisfy the normalization condition $\sum_{i=1}^{M} c_i = 1$, the function ψ actually depends on the concentrations $c_1, \ldots, c_{M-1}$. We use the Greek subscripts $\alpha, \beta = 1, \ldots, (M-1)$, which correspond to this abridged set of concentrations. We also introduce the notation for the partial derivatives: $\psi_{,\alpha} = \partial\psi/\partial c_\alpha$, $\psi_{,p} = \partial\psi/\partial p$.

We recall the well-known relations which satisfy the thermodynamic potential [6]:

$$\psi_{,\alpha} = \kappa_\alpha - \kappa_M, \quad \psi_{,p} = n^{-1}. \tag{A1}$$

With account for (A1) the conditions of phase equilibrium (3) take on the following form:

$$\psi_{,\alpha g} = \psi_{,\alpha c}, \tag{A2}$$

$$\psi_g - c_{\alpha g}\psi_{,\alpha g} = \psi_c - c_{\alpha c}\psi_{,\alpha c}. \tag{A3}$$

Differentiation of $(2M-1)$ independent equalities (6), (A2), and (A3) leads to $(2M-1)$ linear equations which relate $(2M+1)$ differentials of concentrations $dc_{\alpha g}$ and $dc_{\alpha c}$, pressures dp_g and dp_c, and mole fraction of condensate

dW. From these equations we can express the differentials $dc_{\alpha \mathrm{g}}$, $dc_{\alpha \mathrm{c}}$, and dW as linear functions of the differentials dp_{g} and dp_{c}.

To present these expressions in a compact form it is convenient to pass over to the abridged matrix description of the quantities which depend on the Greek indices. So, for the vectors $\eta = (\eta_\alpha)$ and $\eta' = (\eta'_\alpha)$ we determine the product by the formula $(\eta, \eta') = \eta_\alpha \eta'_\alpha$. Moreover, we introduce the notation

$$c_{\mathrm{g}} = (c_{\alpha \mathrm{g}}) \,, \quad c_{\mathrm{c}} = (c_{\alpha \mathrm{c}}) \,,$$

$$A_{\mathrm{g}} = (A_{\alpha\beta \mathrm{g}}) = (\psi_{,\alpha\beta \mathrm{g}}) \,, \quad B_{\mathrm{g}} = (B_{\alpha \mathrm{g}}) = (n_{\mathrm{g}}^{-2} n_{,\alpha \mathrm{g}}) \,,$$

$$A_{\mathrm{c}} = (A_{\alpha\beta \mathrm{c}}) = (\psi_{,\alpha\beta \mathrm{c}}) \,, \quad B_{\mathrm{c}} = (B_{\alpha \mathrm{c}}) = (n_{\mathrm{c}}^{-2} n_{,\alpha \mathrm{c}}) \,,$$

$$b_{\mathrm{g}} = n_{\mathrm{g}}^{-1} + (B_{\mathrm{g}}, c_{\mathrm{g}}) \,, \quad b_{\mathrm{c}} = n_{\mathrm{c}}^{-1} + (B_{\mathrm{c}}, c_{\mathrm{c}}) \,, \tag{A4}$$

$$\Gamma_{\mathrm{g}} = W A_{\mathrm{g}} \,, \quad \Gamma_{\mathrm{c}} = (1 - W)\, A_{\mathrm{c}} \,.$$

We note that both phases — gas and condensate — are assumed to be locally thermodynamically stable; therefore, the matrices A_{g} and A_{c} are positive.

Differentiation of relations (6), (A2), (A3) yields the following equations:

$$W dc_{\mathrm{c}} + (1 - W)\, dc_{\mathrm{g}} + (c_{\mathrm{c}} - c_{\mathrm{g}})\, dW = 0 \,,$$

$$A_{\mathrm{g}} dc_{\mathrm{g}} - B_{\mathrm{g}} dp_{\mathrm{g}} = A_{\mathrm{c}} dc_{\mathrm{c}} - B_{\mathrm{c}} dp_{\mathrm{c}} \,, \tag{A5}$$

$$(c_{\mathrm{g}}, A_{\mathrm{g}} dc_{\mathrm{g}}) - b_{\mathrm{g}} dp_{\mathrm{g}} = (c_{\mathrm{c}}, A_{\mathrm{c}} dc_{\mathrm{c}}) - b_{\mathrm{c}} dp_{\mathrm{c}} \,.$$

Eliminating the differentials of concentrations from these equations, we obtain an expression for the differential of the mole fraction of the condensate:

$$dW = \omega_0^{-1} \, (\omega_{\mathrm{g}} dp_{\mathrm{g}} - \omega_{\mathrm{c}} dp_{\mathrm{c}}) \,, \tag{A6}$$

$$\omega = \frac{(c_{\mathrm{g}} - c_{\mathrm{c}}, (\Gamma_{\mathrm{g}} - \Gamma_{\mathrm{g}} \, (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \, \Gamma_{\mathrm{g}}) \, (c_{\mathrm{g}} - c_{\mathrm{c}}))}{W\,(1 - W)} \,,$$

$$\omega_{\mathrm{g}} = b_{\mathrm{g}} - (\Gamma_{\mathrm{g}} c_{\mathrm{g}} + \Gamma_{\mathrm{c}} c_{\mathrm{c}}, (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \, B_{\mathrm{g}}) \,, \quad \omega_{\mathrm{c}} = b_{\mathrm{c}} - (\Gamma_{\mathrm{g}} c_{\mathrm{g}} + \Gamma_{\mathrm{c}} c_{\mathrm{c}}, (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \, B_{\mathrm{c}}) \,,$$

$$\omega_0 = \frac{(c_{\mathrm{g}} - c_{\mathrm{c}}, (\Gamma_{\mathrm{g}} - \Gamma_{\mathrm{g}} \, (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \Gamma_{\mathrm{g}}) \, (c_{\mathrm{g}} - c_{\mathrm{c}}))}{W\,(1 - W)} \,.$$

The expressions for the differentials of concentrations are obtained similarly:

$$dc_{\mathrm{g}} = (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \, ((W B_{\mathrm{g}} + \omega_0^{-1} \omega_{\mathrm{g}} A_{\mathrm{c}} \, (c_{\mathrm{c}} - c_{\mathrm{g}})) \, dp_{\mathrm{g}} - (W B_{\mathrm{c}} + \omega_0^{-1} \omega_{\mathrm{c}} A_{\mathrm{c}} \, (c_{\mathrm{c}} - c_{\mathrm{g}})) \, dp_{\mathrm{c}}) \,, \tag{A7}$$

$$\begin{aligned} dc_{\mathrm{c}} = (\Gamma_{\mathrm{g}} + \Gamma_{\mathrm{c}})^{-1} \, (((W - 1)\, B_{\mathrm{g}} + \omega_0^{-1} \omega_{\mathrm{g}} A_{\mathrm{g}} \, (c_{\mathrm{c}} - c_{\mathrm{g}})) \, dp_{\mathrm{g}} - \\ - ((W - 1)\, B_{\mathrm{c}} + \omega_0^{-1} \omega_{\mathrm{c}} A_{\mathrm{g}} \, (c_{\mathrm{c}} - c_{\mathrm{g}})) \, dp_{\mathrm{c}}) \,. \end{aligned} \tag{A8}$$

Relations (A6)–(A8) can be used in numerical algorithms for determining the functions $c_{ig} = c_{ig}(p_{\mathrm{g}}, p_{\mathrm{c}})$, $c_{i\mathrm{c}} = c_{i\mathrm{c}}(p_{\mathrm{g}}, p_{\mathrm{c}})$, and $W = W(p_{\mathrm{g}}, p_{\mathrm{c}})$ by the method of small pressure increments in the phases.

Of special interest is the behavior of the differential dW at the points where $W = 0$. For this case, from formula (A6) we obtain the expression

$$\frac{\partial W}{\partial p_{\mathrm{c}}} = - n_{\mathrm{c}}^{-1} \left((c_{\mathrm{c}} - c_{\mathrm{g}}),\ A_{\mathrm{g}} (c_{\mathrm{c}} - c_{\mathrm{g}}) \right)^{-1} < 0 \ . \tag{A9}$$

Due to the continuity, inequality (A9) holds in a certain vicinity of the curve $W = 0$. Thus, in the case of a small mole fraction of condensate, a decrease of pressure in it causes an increase of the amount of condensate (capillary condensation).

Inequality (A9) substantiates the position of the curve LM in the plane (p_{g}, p_{c}). At the same time, this inequality was obtained under the assumption that $W = W(p_{\mathrm{g}}, p_{\mathrm{c}})$ is a smooth function of pressure p_{g}, p_{c}. However, we can show that the function $W = W(p_{\mathrm{g}}, p_{\mathrm{c}})$ cannot be determined as a smooth function for all values of p_{g} and p_{c}.

Indeed, let the pressure in the condensate p_{c} be fixed. The equality follows from (A2) and (A3):

$$\psi_{\mathrm{g}} = \psi_{\mathrm{c}} + (c_{\alpha\mathrm{g}} - c_{\alpha\mathrm{c}})\, \psi_{,\alpha\mathrm{c}} \ . \tag{A10}$$

The right-hand side of (A10) at a fixed pressure and all kinds of concentration is limited from above:

$$\psi_{\mathrm{c}} + (c_{\alpha\mathrm{g}} - c_{\alpha\mathrm{c}})\, \psi_{,\alpha\mathrm{c}} \le \sigma_1 \ . \tag{A11}$$

The left-hand side of (A10) at $p_{\mathrm{g}} = p_{\mathrm{c}}$ and all kinds of concentration is limited from below:

$$\sigma_2 \le \psi_{\mathrm{g}} \ . \tag{A12}$$

Then we use the second relation of (A1) and the property of the Peng–Robinson equation [6] that for $p_{\mathrm{g}} \ge p_{\mathrm{c}}$ and all kinds of concentration the mole density of gas is limited from above, $n_{\mathrm{g}} \le n_*$. Then, when $p_{\mathrm{g}} \ge p_{\mathrm{c}}$, the estimate follows from inequality (A11):

$$\sigma_2 + n_*^{-1} (p_{\mathrm{g}} - p_{\mathrm{c}}) \le \psi_{\mathrm{g}} \ . \tag{A13}$$

Hence the thermodynamic limitation to a value of the capillary jump follows:

$$(p_{\mathrm{g}} - p_{\mathrm{c}}) \le n_* (\psi_{\mathrm{g}} - \sigma_2) \ . \tag{A14}$$

If condition (A14) is violated, Eqs. (A2) and (A3) cannot hold. Thus, with an increase of pressure in the gas and at a fixed pressure in the condensate a certain limit is reached when the problem of mixture decomposition into two phases with different pressures ceases to exist.

This justifies the existence and position of the curve MN. The curve KL is substantiated similarly.

NOTATION

C, maximum value of the jump of capillary pressure; c_{ig} and $c_{i\mathrm{c}}$, concentrations of the ith component in the gas and condensate; c_{i0}, concentration of the ith component in the mixture; f_{g} and f_{c}, relative permeabilities to phase of the gas and condensate; h, thickness of the formation; k, coefficient of absolute permeability; M, number of mixture components; n, molar density of the mixture; n_{g} and n_{c}, total molar densities of the gas and condensate; n_{ig} and $n_{i\mathrm{c}}$, molar density of the ith component in the gas and condensate; p_0, pressure in the formation; p_{cap}, capillary pressure jump; p_{D}, pressure in the gas corresponding to the dew point; p_{g} and p_{c}, pressure in the gas and condensate; p_{w}, pressure on the well face; Q, total flow of components of the mixture; Q_i, total flow of the ith component of the gas–condensate mixture; r, radial coordinate; r_0, radius of the external boundary of the reservoir; r_{w}, well radius; s_{c}, condensate saturation; s_{c1}, threshold of condensate mobility; s_{c2}, threshold of gas mobility; $s_{\mathrm{c,t}}$, saturation after the jump; u_g^r and u_c^r, radial velocity of filtration of the gas and condensate phases; W, mole fraction of condensate in the mixture; κ_i, chemical potential of the ith component of the mixture; κ_{ig} and κ_{ic}, chemical potentials of the ith component in the gas and condensate; μ_g and μ_c, shear viscosities of the gas and condensate; ξ, collective variable of integration of equations; ξ_0, auxiliary variable; ξ_t, point of saturation jump; φ, auxiliary function; ψ, thermodynamic potential (Gibbs energy); ψ_g and ψ_c, thermodynamic potentials of the gas and liquid phases; a, auxiliary parameter; z, auxiliary variable of integration; η, η', c_g, B_g, and B_c, auxiliary vectors; A_g, A_c, Γ_g, and Γ_c, auxiliary matrices; b_g, b_c, ω, ω_0, ω_c, ω_g, σ_1, σ_2, and n_*, auxiliary quantities. Indices: g, gas; c, condensate; cap, capillary; w, well; 0, quantity in the formation; t, jump of saturation; 1, 2, thresholds of saturation; i, concentration of the ith component; r, radial.

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